Bates (H. H.)

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Henry Hobart Bates.

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If there is anything entirely disheartening, it is to see the few landmarks of human achievement disappear before the shifting current of opinion, as headlands disappear under the ceaseless buffeting of the ocean. It is no doubt a matter of poignant regret to the cherisher of ardent theological convictions to see the bulwarks of faith slowly undermined by controversy. So, also, to him who has built his convictions on supposed demonstrable and irrefragable fact, to find nothing unassailable, not even the axioms and postulates conceded for ages as first principles, on which the fabric of science was reared, nor the sublime inductions of Galileo and Newton, on which the modern philosophy called natural—the only fruitful philosophy which man has produced—has been founded.

But the course of criticism shows that there are no first principles. Nothing is unquestionable. Even the mathematic joins hands with the metaphysic. I propose briefly to examine the fundamental grounds of mechanical philosophy, in view of the wide divergence of basal hypotheses in recent years, and especially on account of the importance conferred upon certain speculations by their admission into works of standard reference and authority.*

To do this aright it is necessary to go behind the mere sub-science of mechanics to the essence and substance of things, as did the eighteenth-century philosophers succeeding Newton. The observational data which have accumulated since that time by the splendid efforts of the molecular physicists enable us to review and recast, with some promise, the primary dogmas regarding the physical basis of phenomena. It is legitimate to frame hypotheses on subjects which are still unfathomed, but which confessedly do not belong to the domain of the unknowable. The distinguished example of the authors of the vortex atom would alone justify such a conclusion.

No entirely satisfactory hypothesis of the atom has yet been

* Encyclopædia Britannica, 9th Ed., Articles "Mechanics," "Measurement," etc.

found. I do not design to discuss the vortex atom here at length; for, although it is the most successful form of the Cartesian doctrine of vortical substance, it has not been perfected, and is generally regarded rather as an example of remarkable speculative and mathematical ingenuity, than as a discovery, corresponding with any facts of objective physics. It has insuperable difficulties, some of which have been pointed out by Clifford, and others by Clerk-Maxwell. Moreover, unparticled or continuous substance, the necessary postulate in this hypothesis, is something we not only have no experience of, but find full of inconsistencies with experience, when we gain a clear conception of what it implies. Such a conception fulfills Hegel's paradox that being and non-being are the same, since it forbids all mobility, all differentiation, as was perceived by the followers of Democritus. It simply affords an inviting basis for analytical discussion, on account of the elimination of the very conditions of objective existence which make the mathematical difficulty.

There are some postulates regarding substance which we may probably be permitted to assume at the outset. We may postulate its objectivity, and also its discontinuity. I have no space to review here the time-worn controversy between continuous and discontinuous substance. The arguments, which are exhaustive from the metaphysical side, are as old at least as Democritus and Anaxagoras. Suffice it to say that modern experiential philosophy has decided the battle experimentally in favor of the discontinuity of matter. The dispute only lingers in the region of the atom, where observation cannot penetrate or has not penetrated. The inability to conceive which attaches to all non-experiential affairs is encountered here, coupled with the too great facility of conceiving what is superficially observed, but will not bear analysis. Thus our first impressions of substance are in favor of its continuity. It is only after much reflection that we get the idea of necessary discontinuity; as bound up with the exhibition of existing phenomena. But the wonderful development of the Cartesian mathematics, in conjunction with the infinitesimal calculus, and its great facility in dealing with geometrical continuities, has tacitly revived the Cartesian idea regarding the nature of matter, as synonymous with space relations, which never reached intelligible development at the hands of its author, and wholly declined and disappeared after the

establishment of the Newtonian philosophy, and the discovery of the discrete character of substance.

In point of fact, experience would point to extreme porosity or discreteness as characteristic of substance, rather than to its opposite—perfect continuity. The infinite divisibility of space has nothing in the world to do with the question, though this is a confusion often fallen into. On the contrary, there is an infinite distinction between the infinitesimal discrete units of substance, occupying extension by their interactivity, and the passive infinitesimal resolvability of space continuity. This is the antipodean difference between the Epicurean and the Cartesian conceptions; the former admitting of the operations of force, the free exhibition of motion, the organization of material phenomena, which are phenomena of mobility; the latter constituting a plenum, with only ideal divisions, and phenomenally as necessarily barren a negation as space itself.

Substance is purely experiential. In its essence it is still incomprehensible, because experience has not yet reached down to those recesses. We know nothing of substance except by its manifestations. These manifestations are cognized by us through sense impressions, weighed, compared, adjusted, and analyzed in the mysterious alembic of the mind. First impressions have enormous predominance, and are intensified by heredity of cerebral predisposition and function.

We cognize substance only in bulk by direct perception, and these vast aggregations stand in thought for matter. A drop of water contains incomparably more molecules than the ocean contains drops; a grain of sand more particles than the earth contains grains; and it is this vast mesh of complicated forces that forms the integrated concept of matter to our apprehension. The child, before he can walk, encounters obstacles to movement, reaction to his every muscular effort, of equal measure to his own; and thus his first and profoundest convictions of objective existence are associated with resistance, opposition, repulsion. This impression of matter is so early that it remains with us as its most natural and obvious characteristic.

The idea of weight is also one of the earliest experiences. This idea would not be conceivable to a denizen of the deep sea, for our first ancestor who emerged from the water gained the experience at the cost of great struggle and enterprise. By the natural devel-

opment of muscle and function the child rears itself very early against the constant pull of our pedestal, triumphs over it with new-found energies, dances on tiptoe, and spurns the ground, but is soon content to draw the battle, to wander around a few weary years on equal terms, at length to call in the aid of a stick or crutch, and, finally, to resign the unequal contest, and sink, vanquished and satisfied, to rest in its bosom. Weight thus seemed a natural characteristic of matter until identified and generalized by Newton as a universal and especially a reciprocal property. This generalization transferred the property, in conception, from the naturally heavy body to a cause outside thereof, namely, the earth itself. Here the human mind relucted, for, unlike repulsion, attraction is not an observational fact. All forms of tension, stress, constraint—by whatever name called—are attended in the child's experience with an intermediary connection. The string is necessary to pull the cart, and the action of the magnet upon the iron particles is viewed with astonishment and awe. The sense of mystery does not proceed so far in his case as to contemplate the equally mysterious power which makes his string differ from a rope of sand. The most profound attention of the human mind has not yet fathomed this mystery.

Inertia or mass is a less obvious property, being in early observation and in common apprehension bound up with weight. It was not recognized in philosophy till Galileo's time, nor is it now by the common perception, except after training. A lady makes no scruple of asking to have a loaded car or train or vessel stopped at a given point on the instant, and reinvested with motion any number of times; and would-be inventors often contrive theoretical machines, having numerous heavy reciprocating parts timed to velocities impossible of execution. With beings under other conditions it is wholly different. The sword-fish, *e. g.*, can have no conception of gravity, as he has no perception of it, but his apprehension of inertia is finely cultivated, through the muscular sense, in setting up and modifying the rapid movements in which his existence delights, as well as through his vivid realization of momentum, in the piercing of a whale or a vessel, by which his function is so powerfully exhibited. When once realized by human perception, however, inertia becomes identified with substance as its most primary characteristic.

The old scholastic property of impenetrability, also, is one of the superficial notions of experience, gained in the same way as that of repulsion. It seems to pertain to solids—the typical matter—with approximate accuracy, though calcined plaster of Paris and water, *e. g.*, will occupy a good share of each other's volume, and still form a highly porous solid. But a quart receiver full of hydrogen can have a quart of carbonic acid gas deftly introduced into it as into a void space; and so can a quart of water, at ordinary temperature and pressure, according to Gmelin, without increase of volume, although water is the type of material continuity. As to impenetrability in the molecule we can predicate nothing. The evolution of heat in chemical combinations indicates penetration of volume, with reorganization of the molecule in less space; and there is no reason, except a scholastic one, why two or more molecules, or even atoms, should not occupy the same place, as admitted by the highest authority—James Clerk-Maxwell.

Dimension is also a common notion, derived similarly from supeficial and early experience. Solids alone have figure and assignable dimension, though liquids have fixed volume, and gases variable volume, in inverse ratio to constraint; but even solids are of varying and fluctuating dimensions, according to temperature, density, etc. Solidity and liquidity are, it is well known, but mere transitory conditions of material aggregation, for all matter is capable, by sufficient accession of molecular motion, of assuming that hyperbolic or expansive condition which we call gaseous, and in this state dimension and impenetrability are meaningless terms. Concerning dimension as a necessary attribute of the unit of mass, Clerk-Maxwell says (Encyclopædia Britannica, 9th Ed., Vol. 3, p. 37): "Many persons cannot get rid of the opinion that all matter is extended in length, breadth, and depth. This is a prejudice * * * arising from our experience of bodies consisting of immense multitudes of atoms." That there is no necessary relation between mass and volume as there is, *e. g.*, between mass and weight is shown to common experience by the notably different masses of a buck-shot and a pith-ball of the same dimensions, or of a cannon-ball and a child's hydrogen balloon. A pellet of iridium equivalent in mass to the pith-ball might be microscopic, and, by extreme supposition, infinitesimal. We are not forced, however, to deny to

the unit of mass the finite magnitude, as this would be an experiential fact when ascertained.

The remaining so-called properties of matter are too obviously transitory, accidental, or derivative to require attention. Color, luminosity, opacity, transparency, sapidity, sonority, odor, texture, temperature, diathermancy, plasticity, hardness, brittleness, density, compressibility, conductivity, malleability, fusibility, solubility, and many others, are too clearly but conditions of aggregation, or else mere subjective states due to the way the complicated interactions of the primary qualities affect our senses. What are the primary qualities?

Here is where the modern method of philosophy flags, by the disappearance one by one of the experimental means of approach, as we eliminate the non-essentials. But though the substance is thus elusory, we cannot yet believe it to be illusory.

Chemical and molecular physics have already gone marvellously beyond the ordinary range of sense-perception, by strictly scientific methods. Not only is the discrete character of matter established, but many data of the differentia and organization of the molecule are discovered. Here is a vast field of science in itself. From the ideal molecule, or simple couple, up through the 70 actual organized molecules of our provisional elements, then the chemical molecules of their combinations in vast numbers, discovered and undiscovered, and, lastly, the enormously complex organic molecule in infinite variety, the domain transcends in area for classification that of biologic science. The simple molecule has not yet been discovered, much less the molecular constituent, the atom, or the *indivisible.* It is evident, however, that the properties of matter which are essential, not differential, must reside in the atom. The philosophers succeeding Newton treated the atom and the elementary molecule as one, from lack of sufficient chemical knowledge. We are on a higher plane of information, but their method is not necessarily vitiated by such lack of distinction.

We cannot, as before said, attribute *à priori* to the atom dimension or figure, though we postulate it to aid conception. As the atom is an absolute unit, there is incongruity in finally assigning to it such relative attributes, which are but matters of comparison and degree. There are properties, however, which are inseparable from an absolute essence. These are the properties by which the

essence is manifested to us. We know them provisionally as forces, in the Newtonian nomenclature. Had gaseous matter neither weight nor mass, we could not know of its existence. But these attributes are so constant in matter that we estimate its quantity in terms of them and have no other exact terms. Weight is the statical measure; mass the dynamical measure. And since weight and mass correspond for all substances, under all transformations, we judge that the correspondence identifies them alike with the essence. They cannot be the mere result of organization. They must belong to the ultimate atom.

At this point it would seem proper to attend to a question of definition. Definitions are essential to clearness, on the one hand, and a source of entanglement on the other, if we fall into the scholastic error of regarding a mere word as the coextensive symbol of an idea. Words are evolved during the imperfection of ideas, and language is still a most imperfect medium of expression. Hence, logic is not a science in the sense that mathematics is. I have used the term force. This is a word of much ambiguity of meaning. We may use it as a convenient mathematical expression for a mere rate of change of momentum, or we may go farther and *define* it, as that which changes a body's state of rest or of uniform motion in a straight line; either of which uses restricts it to only a portion of phenomena, and ignores the whole science of statics, dealing with forces in equilibrium and the phenomena of balanced stress. If we give it a more general signification, as that which changes or tends to change, or conserve, the state of motion of particles, or systems of such, either in quantity or direction, we embrace statics as well as kinematics, and get a measurably philosophical definition, if we bear in mind the proviso that we do not thereby postulate force as an entity apart from substance.

And since the compound variable space and time condition which we call motion (of which rest is but a phase) is the sensible resultant of the interaction of such discrete substance by constant rearrangement where readjustment is free, or the potential resultant where confined, we may admit that the observed tension and persistence, of whatever form, is that which effects the phenomenon (though masked by infinite variety and composition), and always across the discontinuity: not as separate entities, but as modes of manifestation of the interacting and pervasive substance itself and

its only manifestations. This we call *force*—the inscrutable agent of phenomena—and this I take to be the true Newtonian conception, as evinced by his maturest conclusions, expressed in query 31 appended to his Optics. (B. 3, 2d Ed., 1717.)

So far as weight goes, it was generalized by Newton to be a reciprocal force or stress, operative without limit on the law which inheres in radial space relations—the inverse square of the distance. The term operative means effective upon mass, namely, bridging the discontinuity. Gravity is the typical attractive force—*vis centripeta*. The relation is mutual by the law of action and reaction, and amounts to a universal tension among particles, controlling all matter everywhere into orderly movements and relations. This is what we postulate from observation, on the Newtonian plan of naming simply what we see. The notion, however, of action at a distance has encountered a metaphysical difficulty in many minds, from the preconception derived from ordinary experience that all affections or stresses must proceed through an intermediary connection, deemed continuous. Even Newton made concession to this prejudice in his oft-quoted letter to Bentley. That there is really no such continuity in any mode of connection known is demonstrable, and the notion itself that the fancied continuity of some rare effluvium could in any way aid the mechanics of the problem is chimerical. Clerk-Maxwell, moreover, has shown (Nature, Vol. 7, p. 324; Encyclopaedia Britannica, Vol. 3, p. 63) that action at a distance is as necessarily implied in repulsion as in attraction, so that theories of repulsion do not aid conception. Ability or inability to conceive, furthermore, is not held even by the metaphysicians to be a criterion of objective truth. Such truths exist independent of the conceiving mind. The conceiving organ was evolved by experience, and conception develops with attention. The first law of motion was wholly inconceivable to the contemporaries of Galileo, and we find such instances even now. Thus, while plain truths are inconceivable until established, some utter absurdities have been deemed conceivable, as, for instance, vacuity of two dimensions. State of mind, then, is no measure of external truth.*

* In this connection, to illustrate how entirely a matter of opinion or prejudice or culture is this notion of conceivability, I quote from a letter

The second force or manifestation of the atom, inertia,—or mass,—unlike gravity, is not unlimited in range of action. As to this property matter is discrete. Mass has both a *locus* and a limit (being apparently dependent for dimension on multiplicity), and amounts to that incomprehensible property by which conservation of motion is maintained. Under gravity, quantity of motion varies according to relations of contiguity, but under inertia motion is conserved in direction and quantity, is modified in direction and quantity by interaction of mass with gravity, and is redistributed by interaction with repulsive force upon an indefinitely near approach of particles, upon conservative principles. Its discreteness gives matter its numerical and finite character, and admits of that interplay which constitutes phenomena.* Its reality and primary

written by Faraday to Dr. Playfair, in response to some inquiries of the latter about his atomic opinions:

* * * "I believe in matter and its atoms as freely as most people—at least, I think so. As to the little solid particles which are by some supposed to exist independent of the forces of matter, and which in different substances are imagined to have different amounts of these forces associated with or conferred upon them, * * * as I cannot form any idea of them apart from the forces, so I neither admit nor deny them. They do not afford me the least help in my endeavor to form an idea of a particle of matter. On the contrary, they greatly embarrass me; for, after taking an account of all the properties of matter, and allowing in my consideration for them, then these nuclei remain on the mind, and I cannot tell what to do with them. The notion of a solid nucleus without properties is a natural figure or stepping-stone to the mind at its first entrance on the consideration of natural phenomena; but when it has become instructed, the like notion of a solid nucleus apart from the repulsion, which gives our only notion of solidity, or the gravity, which gives our notion of weight, is to me too difficult for comprehension; and so the notion becomes to me hypothetical, and, what is more, a very clumsy hypothesis." (Playfair's works, Vol. 4, p. 84.)

Here we see a difficulty opposite to that usually encountered, for, while many people profess an infirmity of conception of the forces apart from the imaginary vehicle, Faraday finds the vehicle of no use as a carrier of the properties, but a positive impediment.

* This property has a multiplicity of names in the Newtonian nomenclature, according to the varying aspect of its function. Thus, in the aspect of persistence of mass in state of rest or of motion uniform in direction

character, when once apprehended, have proved more acceptable to the imagination than has the conception of central force, and under appulsion hypotheses (with the aid of that other readily accepted property, repulsion, and certain highly artificial hypothetical media), it has been made to do duty in providing so-called explanations of gravity, under its form of *vis viva*.

It has always seemed to me that the mode of approach adopted by Boscovich was the most philosophical and rigorous of any. He viewed matter for the purposes of mathematical treatment and for investigation of its essentials, as divested of accidental and fugitive properties; and as the analytical calculus had not then become so developed as to wholly fascinate the attention of geometers with abstract and ideal relations, he proceeded from prime physical data. He thus identified matter by those apparently general and characteristic properties recognized by Newton as the basis of mechanical philosophy in conjunction with the laws of motion. These properties are, as before said, gravity, inertia, and repulsion; or, as characterized by function, attraction, conservation, distribution. In this view matter consists of certain *loci* of central forces, mutually attractive by the first property according to a variable law in the duplicate inverse ratio of distance without limit, but restricted in manifestation as to the second property to the infinitesimal *locus*, thereby excluding unitary dimension. Contemplating matter under this aspect alone, a dilemma arose. For gravity waxing by the law of inverse squares of the distance up to the focus or origin involves the consideration of infinite force and apparently of infinite velocity in the limit, in the supposable case of rectilinear ap-

and quantity, *i. e.*, of resistance to change of state except in conformity with motion impressed, the property is called *vis insita*, which may be *vis insita activa* (momentum), or *vis insita passiva* (*vis inertiæ* of mass.) In its aspect of acquirement of a new state of motion by interaction with other forces or masses, Newton called the new state thus superposed *vis impressa*; which, when the operation of acquirement has ceased, becomes again *vis insita*. In its aspect of persistence of mass towards uniform direction of motion under the constant deflective stress of vector central force, it is called *vis centrifuga*. And in its active form, conditioned by motion acquired, its capacity for furnishing motion from its store, either for impressing motion upon other mass, with consequent loss, or for supplying the potential fund under the drain of adverse central force, is called *vis viva* (energy.)

proach, at which point the equations become unexplainable. While Euler and La Place differ in their interpretations of the result, Boscovich sought to solve the apparent absurdity and inconceivability by the invention of his ingenious and complex system of alternate spheres of attraction and repulsion, or change of sign, on a very near approach, with infinite repulsion at the focus, which so loaded down and vitiated his hypothesis as to cause its rejection. This result was similar to that of Le Sage's speculations and those of the Ptolemaic astronomers, each thus working out the falsity of his respective scheme by superadded complications to readjust the theory to the progress of criticism or of observed fact.

By attributing finite magnitude to the atomic mass, however, Boscovich's difficulty disappears, as I had the honor of pointing out before this Society some ten years ago. This may be deemed a violent hypothesis in regard to a positive discrete simple absolute, as the atom is presumed to be, but parallel difficulties inhere in any other finite supposition, as, *e. g.*, a sphere of repulsion. Under my provisional assumption, the way out follows from an elementary proposition of Newton's, and it does not demand the gratuitous change of law or of continuity involved in the resort of Boscovich. The movement of a gravitating particle under stress of a center of gravitative force would be in all respects as the great 18th century mathematicians have demonstrated, until the margin of the particle reached the attracting center, where, if we suppose the attractive virtue to prevade the particle equally throughout a certain finite volume of mass, however minute, as gravity does the mass of a sphere, the maximum of attractive force would be attained; for, as Newton has shown, homogeneous spheres are controlled under gravity by a law of force varying directly as the mass and inversely as the squares of the distance between their center of mass and the attracting center, at all points *beyond* the surface, and directly as the distance between the said centers *within* the surface; so that, after passing the surface, the attractive center must proceed onwards to the gravitating center of mass (relatively), not by a force increasing to infinity, but by a force decreasing to zero, after passing the maximum, since it is balanced at the center by opposing stresses.*

* Let M be an exaggerated particle of mass and C a fixed center of gravitation external thereto. Newton proved that for all positions outside of a

A similar law of attraction prevails between two gravitative particles when both are similarly endowed with finite spherical volume and mass, excluding the idea of impenetrability (which is not a necessary attribute of mass), the Newtonian law being the product of the masses divided by the product of the distances $\left(\frac{Mm}{dd}\right)$* for outside positions.

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gravitating homogeneous spherical mass the stress is precisely as though the whole mass thereof were concentrated at the center of said sphere, and varies directly as the mass and inversely as the square of the distance between the said center and the fixed center of gravitation; *i. e.*, $G \propto \frac{M}{d^2}$. The maximum of gravitating force will here be at the surface, where d is minimum. He also proved that at all points within a homogeneous gravitating spherical concentric shell a gravitating particle is uniformly affected by balanced attractions. Hence, the stress for any smaller concentric sphere is $g \propto \frac{m}{r^2}$, m being the smaller spherical mass and r the reduced radius. But since homogeneous and similar masses are as the volumes, and similar volumes are as the cubes of the homologous dimensions,

$$m \propto r^3 \therefore g \propto \frac{r^3}{r^2} \propto r.$$

The maximum of gravitating force is here also at the surface, where r is maximum.

* I write the formula this way because it is possible that we have been in error all along in regarding the denominator as a radial space relation, as implied when we write it $\frac{Mm}{d^2}$. In discussing the deflection of the particle under gravity, Newton, for mathematical simplicity, treated it as governed by a fixed attracting central force, and in testing various relations found that the radial space relation gave the true path of the planetary bodies under the immense preponderating influence of the sun's mass. The fixed center of attraction is, however, a mathematical, not a physical, condition, and can only be realized by making $M = \infty$, when we get a form of expression which does not give a law of force. I think it possible that the relation is a mere reciprocal distance relation, since the stress is mutual for the masses and each is equally distant from the other. The inverse form of the relation, moreover, may arise from our subjective way of viewing distance, as measured outwardly from ourselves, since we have to go from here to yonder. It is possible to look upon the relation as really one of contiguity or nearness, and by placing $\frac{1}{d} = c$ we get the cosmical law of gravitation as $Mcmc$. This, however, would not be a useful formula, since we are not accustomed to expressions which attain maximum value with minimum magnitude

For positions of encroachment the law is more complicated, and forms an interesting field for mathematical discussion. Where three or more atoms are superimposed the problem becomes too complex for discussion. It is noted, however, that such compound atom, if quiescent from extreme abstraction of heat, would be in a condition of elastic equilibrium, ready to respond like a bell to the slightest disturbances. In all these cases of interp~~ret~~enetation the law of stress would be finite and diminishing, and if the line of encounter should chance to be a right line through their centers (a condition rare ~~in inverse ratio to sectional area~~), they would continue on or repeat according to energy of approach: while upon any other lines of approach orbital relations would supervene, in modified curves of the second order, either hyperbolic, parabolic, or elliptic, according to velocity, and with or without partial penetration, according to nearness of approach.

Boscovich, however, did not adopt this solution, although within his reach. The problem of the action of a gravitative particle as controlled by an attractive center has several aspects of statement, which may be confined to four, for practical investigation. In the first, where the particle is assumed to be without mass, no discussion is possible, for the two suppositious points instantly assume the same locality, and end the relation. In the second, where the particle is endowed with inertia but not magnitude (and the attractive *locus* fixed by postulate), the element of motion enters, but infinite terms appear in the equations in the limit, forbidding interpretation. Thirdly, when we attribute finite magnitude to the gravitative particle for gravitative pervasion, as in actual spherical masses, no infinite terms appear, and we get an intelligible mathematical discussion, with planetary results for exterior positions, and pendulum results for interior positions, as I have heretofore demonstrated; and lastly, when both the gravitating *loci* are invested with similar attributes of volume and of mass (excluding extraneous notions of ordinary collision and repulsion from the problem), the results are similar to those of the third hypothesis. I do not introduce any of the mathematical discussions here, as the dynamics of the particle have been fully treated by mathematicians, though I am not aware that any of them have pursued it to physical conclusions.

It is not likely, however, that there is any matter so simple as this modified Boscovichian atom; that is, which can be identified.

All the matter we know of is already compounded and highly organized. The ideal simple molecule would consist of a single pair of such atoms, bound to each other in orbital relations of more or less eccentricity, including the extreme rectilineal form of simple pendulum-like oscillation through one another's centers; and it is a most significant fact that spectroscopic observation of all incandescent matter shows atomic matter to be in this state of transverse or orbital oscillation with inconceivable but synchronous rapidity without regard to range, according to the pendulum law of stress varying directly as the range of oscillation, discovered by Galileo. Any theory of the simple molecule must take cognizance of this observed fact. Another cognate fact is that the law of elastic cohesion manifest in all elastic tensile action—"*ut tensio sic vis*"—is a parallel law of stress, as illustrated in the spring balance weighing scale, the spring dynamometer, the isochronous spring governor, etc., and is a function of molecular and ultimately of atomic force and distance.

If the atom is really thus characterized, the repulsion or resistant property experienced in matter becomes worthy of investigation, since it drops out as the primitive affection or disaffection postulated by Boscovich. I have shown that it is not necessary to oscillatory motion. We must admit that the notion of rebound or recoil, in the ordinary sense, between simple atoms possesses difficulties. No less does the idea of plasticity or destruction of momenta. Consider what is involved in the hypothesis of two absolutely hard, rigid, unparticled, homogeneous spherical bodies of any magnitude at all, if possessed of mass, meeting on a rectilineal central line of motion. We know what would happen in case of ordinary spherical elastic masses or aggregations of molecules. Such merely undergo, first, apparent contact, then compression, deformation, strain, accumulation of stress, retardation of velocity, momentary arrest, acceleration on new lines of departure, relief of strain, recovery of form, redistribution of momenta, and final resumption of uniform velocities, with relative motion inverted and aggregate energy of motion unimpaired, unless permanent distortion and heat have absorbed a portion. All this complex action is involved in the term elasticity. None of this could take place with simple undifferentiated particles, unless we invent for them a mystic atmosphere or cushion of repulsive capacity surrounding the *locus*, as Boscovich was forced to do

by logical conclusions. Without this, contact would be absolute and instantaneous at first impact. As hardness involves impenetrability, absolute destruction of motion on the instant must ensue; that is, motion and no motion at consecutive instants of time; a discontinuity unknown to experience, and known to be inconsistent with the nature of motion and of time. This argument from breach of continuity is due to Leibnitz. Conversion into heat motion is excluded, heat being a mode of motion of the entire atom. Moreover, the destroyed motion has to be recreated instantaneously in new directions, for destruction of energy cannot be postulated. This geometrically angular motion is also unknown to experience, for all deflected bodies pass by continuity from motion in one direction into a new direction, and, so far as we can see, must do so. These discontinuities in translatory relations are therefore put aside, not because they are inconceivable, but as illogical and non-experiential. Simple repulsion by contact without occult intervention is a false suggestion, and we find that we get the pseudo-conception from our false observation of what occurs in the collision of sensible masses, somewhat as we make a false observation and generalization about material continuity, or about tension, from a superficial perception of matter; thus creating concepts from supposed experience which can have no true objective counterparts. I shall recur later to a possible derivative basis for repulsion.

It is remarkable that to Newton we owe the final establishment of the majority of those fundamental and universal truths which by simplicity and generality seem to touch the absolute; that is, more than to any and all other philosophers combined. Thus, of the six ultimate generalizations, four were formulated and placed on an impregnable basis by Newton: the three laws of motion and the law of gravitation. All of these were inconceivable when first promulgated, were hotly controverted on the metaphysical plan, were finally established experientially, and are now generally accepted as axiomatic by the modern mind, except for sporadic reversions which appear now and then to deny their actuality and reassert their inconceivability. The remaining two universal inductions are the collective group of axioms formulating the relations of extension—the only enduring remnant of the Greek philosophy—and the law of the conservation and unity of energy, unperceived in Newton's time in its generality, though taught as a dogma by the

Cartesians. These also are still held to be inconceivable by certain disciples of metaphysical methods and axiomatic by others. Such mental attitudes should lead us to believe that simplicity has been arrived at in all these cases and the boundaries of explainable knowledge reached, where inconceivability necessarily begins.

It has been said that paradox is born either of confusion of thought, or of knowledge, or confusion of statement arising out of the imperfection or subtlety of the verbal vehicle of thought. Thus, as Clerk-Maxwell points out, the celebrated arguments of Zeno of Elea, establishing the inconceivability of motion, represented in the paradox of Achilles and the tortoise, were unanswerable and unanswered until Aristotle showed, some half century later, that duration is continuous and incommensurable by numerical methods in the same sense that extension is. The old logical dilemma of the irresistible force encountering the immovable body was insoluble to the Greek mind, both from lack of physical knowledge and lack of verbal clearness of statement. The acute sophist knew not the nature of force, the constitution of bodies, the conservation, transformation, and dissipation of energy, and consequently knew not the refuge and escape from the dilemma contained in the perception of the conversion of molar energy into heat energy, expansion, and dissipation. The resources of verbal subtlety and of inner consciousness failed, as they always do. Something of the same difficulty remains in modern problems, where observation and strict verification are, from the nature of the problem, inapplicable, or where the confusion arises from the still-existing imperfection of language, or, again, where generalizations, both clearly made out and clearly formulated, have not passed into the instinctive popular apprehension. The modern dilemma of the inconceivability of infinite or finite space is, I take it, due to the metaphysical form of the statement. For when we reflect that the ideas of immensity and of infinitesimal resolvability are but abstract generalizations of the merely relative continuities, extension, distance, and dimension, which are in their turn but abstractions of the sense-perceptions, form, translation, and volume, the statement becomes intelligible and entirely conceivable, and I think, though with deference, saves geometry; that is, the universality of that system of inductive postulates regarding the relations of extension and inferences therefrom, known as geometry to the Greek philosophy.

but now named Euclidean by certain analysts whose so-called geometry is symbolic. Geometry is therefore able to deal with all aspects of extension, witho[illegible] regard to limit, in spite of some infirmity in the Greek method, for scale cannot affect the generality of extension relations, and abstract unconditioned space is not an entity but a mere negation, concerning which relative propositions are unintelligible. A false philosophy regarding space is at the root of all modern heresies concerning geometry and mensuration, founded in misapprehension of the Euclidean inductions or generalizations.*

The first law of motion is but the formulated recognition of inertia, which is only manifest in conjunction with motion, actively or passively. It was known to Galileo, and laid down by Descartes as a law in his Principia. It is a cosmical truth, bound up with the absolute nature of mass and the true relations of extension, which correlates the whole fabric of dynamical knowledge with rectilinear geometry, curvilinear motion being demonstrably not a simple state of conservation under inertia, but a resultant of multiple forces. The simple action of mass under the first law of motion, if undisturbed, furnishes the absolute unreturning rectilineal path which overthrows all speculation about possible ideal spaces. I here recall a book written by a learned American of Philadelphia—learned, that is, according to the mediaeval standard of the colleges—and published only during the past year, en-

* There are two opposite though similar forms of error in the assumptions regarding space. The first is that space is a specific or perhaps generic entity or objectivity *per se*, possessed of conditions and attributes, like substance, such as dimension (in several), differentia in locality, figure, as curvature, etc. (hence necessarily finite), and only uncognizable by us simply for lack of perceptive faculties to correspond. This is the fundamental error, as it seems to me, of Riemann and Lobatschewsky. The second is that of the older Cartesians, who viewed space as but the mere attribute or synonym of substance, and inconceivable apart from it, so that bodies separated by void space would be absolutely in contact without regard to distance. Both of these speculations are purely metaphysical, and non-experiential, the latter resulting from the old scholastic method of syllogistic deduction from primary postulates of verbal definition, and the former from similar inferences from the forms of the analytical logic of symbols, the use of which is still in the scholastic stage. Like Zeno's paradox, these merely intellectual difficulties should be removable by intellectual processes.

titled "An Examination of the Philosophy of the Unknowable, as expounded by Herbert Spencer," wherein he naively lays down the first law of motion as unintelligible except by appulsion. Motion, he says, in the absence of propulsion is inconceivable. I have no space here to reproduce the explanation evolved out of consciousness by this reasoner to account for the action of a ball struck by a bat after leaving the bat. It resembles in ingenuity and gratuity some of the inventions devised to explain gravity. The notable thing about it is that here, at this date, is a mind of good caliber, informed in the higher schools of learning, which is still of the mental period of Aristotle; a mind which has evidently never apprehended inertia, nor heard of the great contributions to knowledge made by Galileo and Newton, by which philosophy was entirely revolutionized.

The second law of motion, regarding the independence and coexistence of motions, on which we occasionally see comments in the metaphysical vein controverting its possibility, has long been established experientially. Its early experimental proof is attributed to Galileo. Yet I recall a pamphlet written and published only during the last year by a learned German at Leipzig, the theme of which was that "the sun changes its position in space, therefore it cannot be regarded as being in a condition of rest." This, he concludes, overthrows the entire fabric of Copernicus, because the planetary orbits in such case cannot be closed.

The third law of motion is but formulated reciprocal stress, in its modes of compulsion and repulsion, through which mass acts on mass to redistribute motion by what appears to be necessary law. The stress is necessarily reciprocal, since there is no *point d'appui*, or fixed fulcrum, in the universe.

We have thus been brought to the boundary of the absolute, where all is inconceivable until found out, and where the simple data are unexplainable. All examination seems to continue to point to mass and weight as the ineffable simple insignia of substance standing on this limit. We must accept something as elementary fact; what shall we find more elementary? Repulsion is still debatable; for, if we make an issue between repulsion and compulsion as contradictory primary attributes of the same essence, or untenable in conjunction for artificiality, by far the greater difficulties attach to the former, some of which I have already alluded

to. The profound mind of Boscovich was forced to accept repulsion as a primal quality, but in deference to the physical hypotheses of his time, he overloaded it with complication. This has been weighed in the balance of philosophical judgment and found wanting. I have intimated that there are possible grounds for surmising that it may not be a simple property of the atom, but a mere mode of distribution of energy dependent on composition of motion of atomic mass after change of sign, *i. e.*, a mode of *vis impressa* after exhaustion of the space relation; for, mathematically, the hyperbolic lines of approach and recession of two atoms under the high proper motion characteristic of the atom, and on lines not directly central, would be similar, at sensible distances, in their asymptotes (which would be the practical paths), whether the deflection were due to attractive or repulsive stress, though acceleration and retardation at the passage of the infinitesimal focus would be inverted.*

* It is well known that for any finite system of two particles controlled by gravity the lines of movement are closed curves of the second order, of more or less eccentricity, about the common center of gravity, which, for equal masses, would be midway. For an infinite system under the same conditions the orbits are parabolic, but for a system to which the particles enter by extraneous motion the lines of movement are hyperbolic, thus:

Fig. 1

Now, under repulsion, the lines of motion are seen to be similar, A B, D E, Fig. 2, being asymptotes of the hyperbolas representing the two paths at sensible distances:

It therefore seems to me immaterial to result which of the two modes of passing the infinitesimal focus is the true one. In either case the distance at passage is infinitesimal, and the force may be as near infinity as the facts require it to be assigned. The normal or rectilineal encounter is here excluded from supposition. In that case, under repulsive stress, as postulated by Boscovich, the recoil would be rectilineal and opposite, without breach of continuity. Under attractive stress, with finite volume of the atomic mass, penetration would ensue as before shown; but without dimension or repulsion we have an insoluble condition, although the occurrence would be infinitely rare. Only one pair of elements is here considered. In all real encounters, whether of masses or molecules, the effect is a vast resultant, but should not be different in kind from that of the elements; that is, hyperbolic or expansive between alien systems under motion. As the number of elements ordinarily engaged could not be represented by any numerical places of arabic notation for which we have names, we see the hopelessness of stating the problem mathematically. I therefore do not presume to

FIG. 2.

This encounter represents only one element of the molecule, of which myriads are engaged at every recoil of molecules, not to speak of solids. It is thus seen that the mesh constituting the molecule is ordinarily impenetrable to other meshes. If the curve F G be allowed to represent the outline of the molecule, the limb of the solid to which it belongs, say a buckshot, will be represented by the Sierra Nevada, or the Andes, and its diameter would be measurably represented by that of the earth, as approximately shown by Sir Wm. Thompson in the case of a drop of water.

offer this as an explanation of repulsion, and I confess that to me repulsion is in its mechanism incomprehensible. We know the result experimentally, and that is resistance to penetration, and reaction at insensible distances on an undefined boundary which begins prior to contact and increases in a high exponential ratio as approximation progresses. The contact boundary of any solid—even the smoothest and hardest—resembles the astronomical limb of Jupiter in geometrical indefiniteness. The contact transmitter in the telephone, the whole range of whose phenomena occurs under pressure and so-called contact of varying degrees, illustrates how relative a thing is contact. Under high velocities the distinction between solids, liquids, and even aeriform bodies entirely disappears in respect to repulsive reaction, though this is the most sensible distinction between them under low velocities.

We may, therefore, adopt the conclusion that if any of the apparently simple properties of the atom are to be thrown out as derivative and secondary, presumption points to repulsion as the complex one. We could possibly account for phenomena in a universe bound together by purely tensile stress, but most of the sensible phenomena of solids—cohesion, affinity, tenacity, etc., including nearly all of statics—remain hopelessly unattackable problems under a hypothesis of pure repulsion, like that of Le Sage, or Preston. It is to be noted that the kinetists who freely postulate repulsion and appulsion, without analysis, as a primordial fact, but reluct against compulsion or tension, are forced to the invention of the most complicated and gratuitous mechanism and media to explain the phenomenon of gravity, and then without attainment of result. Le Sage's atom is too complicated, even without his supposititious or extra-mundane operative machinery; and the vortex atom is but a mere analytical expression for an unproducible condition in a figmentary mathematical plenum.

The thesis that conservation is the characteristic by which we identify objective existence will not bear the test of examination. It is only in the most recent times that such a quality has been known or imagined, and its establishment, both as to matter and energy, is justly viewed as the triumph of modern philosophy. The evocation of matter from nothing and its relegation to nothing, even by the finite will of a wizard, was ever a common and universal notion, which did not at all impair the belief in its present reality

and substantiality. We have only to go to Apuleius for this, and it is doubtful if even now the notion of the indestructibility of matter is anything but a scientific conviction, for do we not see numbers of our contemporary fellow-citizens meeting together frequently in our midst to witness feats of materialization out of nonentity by powers akin to those of the sorcerer, without an idea of incongruity? Nor has the essentially modern doctrine of the conservation of energy anything to do with the belief in its reality. Few people apprehend it even now. No philosopher understood it a hundred years ago. Its verity rests on a sufficiently general inductive basis, from the refined and exhaustive experiments of Joule, and the theoretical conclusions of Mayer and Clausius, and it is accepted in the same sense that the law of gravitation is accepted. But the duality of matter and energy to the exclusion of force is a verbal shift, the assumption of which removes no difficulty. Matter, the object, remains unexplained; and energy, the phenomenon, becomes segregated and unintelligible. Energy, in fact, is but mass in phenomenal manifestation, being a product of triple factors, two of which—translation and speed—are not things, but variable and evanescent conditions, and, taken together, constitute motion. Mass is the absolute or persistent factor, but the evanescent character of the variable component—motion—would render the entire phenomenon—energy—apparitional, were it not for the distance relation involved in motion, which, under the same inscrutable agency which modifies and saps the motion renders it potential upon change of sign. This agency, the dynamical source of the manifestation, being central to mass and likewise persistent and constant, renders the positive and negative potentialities of movement constantly equal, and the actual and potential energies consequently complementary, from which energy gets its character of conservation.

Energy cannot therefore be that other reality of existence (besides matter), since force is clearly the one reality at the bottom of the manifestation of both, to whose persistence and resistance to change, except through transformation, the conservation of both is due. This one reality is, in its triple aspect of causation, (1) attraction—the source and modifier of motion; (2) inertia—the conserver of motion; and (3) repulsion—the distributer of motion; or, more correctly, in its aspect of quality: (1) *vis centripeta*—the power of mutual control across distance; (2) *vis insita*—the power

of persistence in state of motion impressed; and (3) the distributive power of imparting and acquiring motion by transfer, at minimum distance, which may be called *vis partitiva*, the result of which is Newton's *vis impressa*. Matter thus comes into the world of phenomena by the simple presence of other matter, permitting the exhibition of these comparisons and interactions, involving the conditions of contiguity, distance, position, translation, direction, succession or sequence, and time-rate for the continuous increments, decrements, successions, and uniformities, all bound up in the compound variable continuity—motion. With motion and distance comes the dependent phenomenon—energy—active and potential, which should be a constant, the numerical units of mass being constant throughout immensity, provided the sum of the motions, potential and actual, be constant. This the dynamical theory deduces from the fact of central force (for without force potential motion is ridiculous), and the thesis of the conservation of energy is a dynamical truth or nothing. It is therefore all the more extraordinary that certain kinetists, who reluct against central force, should have selected, out of all the manifestations of the universe, the variable and conditional product—energy—to be the one reality or objectivity, aside from the undefined hypostasis—matter—as a primordial simple fact at the basis of phenomena. It has been mathematically demonstrated by Mr. Walter R. Browne (London Edinburgh and Dublin Philosophical Magazine, January, 1883, p. 35) that the conservation of energy is true if the material system is a system of central forces, and is not true if the system is anything but a system of central forces. In fact, the ordinary theoretical proof of the principle of the conservation of energy assumes the forces acting to be central forces, *i. e.*, reciprocal stresses between units of mass, as recognized by Clausius in his Mechanical Theory of Heat. Moreover, the entire body of kinetists, who have aimed to supersede gravity or central force, have freely assumed an extramundane supply of motion and energy without regard to conservation, and it is notable that every hypothesis for this purpose yet broached involves the constant expenditure of work without recovery, and postulates the accession of energy in infinite influx from some occult source, of which only a small portion relatively is available or manifest in observable phenomena, thus violating all three of the canons of philosophical ascription—true cause, sufficient

cause, and least cause. Such is the power of conception of the unknown in endeavor to explain the inconceivable known.

If the dynamic hypothesis of perpetual transformation of energy could be established as a universal induction, with as much generality, *e. g.*, as the statement of the law of gravitation, it would establish and confirm that law, by Mr. Browne's demonstration, as something more than a law, to wit, the necessary constitution of matter as a system of central forces and nothing more, substantially as conceived by Newton and elaborated by Boscovich. At present it is but a dynamic induction, but the theory of gravity is no more. Our appliances are material, and we can deal with molar forces, but only indirectly and inferentially with those which are atomic. Conservation is indubitably true of energy in the mechanical and molar sense, under the laws of dynamics and the persistence of force. It is, also, experimentally true, so far as we can trace it, of those less understood forms of energy which are molecular or atomic, the establishment of which was the great glory of Benjamin Thompson, Clausius, and Joule as to heat, and of a multitude of observers as to electrical energy. We infer it as a general truth of these energies (formerly known as imponderables, since they are not manifestations of matter in the concrete), from the fact of their convertibility with other modes of energy which are undoubtedly dynamical, and also from the intimate connection of electrical energy with one of the specific exhibitions of central atomic force—magnetism. Such clews create a warrantable presumption that the phenomena in question will all ultimately be classified among the modes of atomic mass and motion, inductively as well as hypothetically. Possibly in the investigation of these evanescent modes of energy the missing simple particle may come to light. Provisionally, we are entitled to rank them among the mechanical modes of energy, as products of the same material forces, assuming, until the contrary is proved, that some form of matter is concerned in manifestations so correlated by conservation with undoubted material activities.

In including the imponderables within the general dynamical law of conservation, we have to take account of the phenomena of dissipation, first pointed out by Sir William Thompson. It is true that heat (as well as electrical energy) is strictly correlated with and interconvertible with energy of mass motion, as before

stated, but in its final form energy seems to take leave of matter altogether, so far as our perceptions can follow it, and disappear as a material phenomenon (though liable to reappear wherever matter is encountered whose particles are deficient in a like species of atomic motion with that which disappeared; which fact indicates that atomic mass is still a factor, with its inherent property of persistence and transference). The earth and all upon it is radiating heat energy away into space at the constant rate of 500° F. of absolute temperature, more or less; the sun and the visible stars at the rate of many millions of degrees. Much energy also passes off in the luminous form. Of electrical and actinic energies we know less, and of some we doubtless know nothing. This amounts to a constant drain of the dynamical supply of energy. These final forms, the radiant energies, have a remarkable specific high cosmical velocity of their own, which is a function of something not material, or at least not molar. It is supposable that, in addition to the dynamical source of motion from central forces, and the contraction of systems in dimension which supplies dissipation, there may be an inherent and primordial store of atomic motion. The high proper motion of some of the stars, beyond what can be accounted for on dynamical principles, and the inexhaustible and enormous supply of radiant energy from the visible stars, have afforded grounds for such a surmise, but these speculations do not belong to the domain of mechanics.

And here we must bear in mind that the dynamical theory, in placing these assumed agencies and modes of interaction in causal relation to phenomenal motion, by no means predicates or can predicate anything concerning absolute motion or its cause. The lack of this distinction may have proved a stumbling block to some in comprehending the idea of force. Were it not for the observed dissipation of energy no system could become contracted in dimensions a particle by the interactions of material forces, nor is there now any known way by which the material system can be expanded in dimensions except by the accession of motion from extra-mundane sources, which there is no scientific mode of ascertaining. The sum of motions under the action of forces remains the same, and any change would imply creation or annihilation, which is not ascribable to a material agency. Primordial dimension remains as inscrutable a fact as ever, and primordial motion an unsolved problem.

In conclusion, I know nothing of force except as a manifestation of matter, and nothing of matter except through its manifestations. It is substance that interacts with substance, so far as we know, always reciprocally, and force is but the convenient translation of the terminology invented by Newton to designate these several species of modes of action, in the word *vis*, with its appropriate adjective. He was arraigned by the Cartesians (and virtually is by their modern representatives) as the reintroducer of occult qualities into philosophy, but his statement was "*hypotheses non fingo*," and to a similar charge brought against him by Leibnitz he pertinently replied that it was a misuse of words to call those things occult qualities whose *causes* are occult though the qualities themselves be manifest.

I have adopted gravity as the type of central inherent force—*vis centripeta*—but I would not thereby be understood as excluding from the category of material forces any and all other modes of tensile or constraining force which may be hereafter made out as specific, by the elucidation of such phenomena as affinity, cohesion, tenacity, elasticity, ductility, viscosity, capillarity, polarity, magnetism, etc., now so little understood, any more than I would exclude any form or mode of energy which may be observed, from the category of material phenomena. The Newtonian doctrine of force would not be impaired by such discovery, and its strength lies in the fact that it as readily includes static phenomena—that despair of the kinetist, who has no imaginable hypothesis by which to range them under a form of motion—as it does kinematical phenomena. Statical force (Newton's *vis mortua*) cannot be ignored in a theory of force. The straw that breaks the camel's back—the very lightning that crashes through the sky—are familiar examples of its power made manifest. Its reality may be exemplified by suspending two heavy balls of equal weight at equal heights—one by an elastic cord, and the other by a tense string. The difference of effort required to displace the two vertically upwards, which can be measured, makes sensible the difference between the two forms of balanced statical forces. In the one case the antagonizing force is suddenly withdrawn, and in the other gradually. Wherever strain exists—and it is everywhere—there force is as certainly present as when it becomes manifested in a stress relieved by motion and measurable in terms of energy.

Let us, then, give up the standard of *a priori* conceivability, in view of its many historical failures, and adopt as possible that which is provisionally ascertained. The "ego" and the "cogito"—Cartesian starting points—have proved barren and irrelevant in Philosophy. True Philosophy is concerned with objectivity. The data of consciousness, mainly acquired in infancy or in the womb, are blind guides. Many an ego, whose brain was his cosmos, has run through his brief subjectivity, but the order of nature endures. The same facts are continually observed, verified, recorded, and rectified, but the observers change. Their intelligent observations add to the sum of knowledge. This is all the proof we need of objectivity, and all we will get. The insoluble difficulties of Philosophy have disappeared one by one since the happy thought of eliminating them by observation entered. The immortals are those who have successfully applied this method. It is only where observation fails that the insolubility lingers. Beyond the sphere of the knowable it will continue, in spite of introspection. How masterful is fact in the presence of the most intricate mental subtleties. The ball leaves the bat, in spite of the inconceivability. Galileo's plummet dropped from the moving mast strikes the deck and not the water, in spite of the inconceivability. The Earth returns in its orbit, to the second, in spite of the sun's rapid fall through space, and of the inconceivability. Two opposed horses can pull no more than one, in spite of the inconceivability. The guinea and the feather dropped in the exhausted receiver strike the plate together, in spite of the inconceivability. The isochronous pendulum swings through the widest arc in the same time as through the smallest, in spite of the inconceivability. The minute hand overtakes the hour hand, in spite of the inconceivability. The magnet draws the iron with undiminished force through all possible interpositions, in spite of the inconceivability. Could an exception be found, the perpetual-motion "crank" would work a greater inconceivability, by the instant contrivance of a power-generating machine.

We need not aspire, therefore, to remove any of the inconceivabilities of the external world. We must accept them as natural to the finite comprehension, as necessary to faculties which act by comparison, and above all as evidences of objectivity. On the other hand we should avoid that opposite error of the introspective

school, of deeming that probable, or in any way connected with fact, which merely seems conceivable. I have shown that while the simplest truths have generally proved inconceivable until found out and established by genius, the greatest absurdities have had ready currency without a doubt of their conceivability. This all mythology shows. Such rubbish as "a thing cannot act where it is not," and "a body cannot move where it is not," or "a cause cannot precede its effect"—mere metaphysical assertions or subtleties in face of everyday fact—were stumbling blocks for ages. Such assumptions formed the basis of deduction in lieu of observation, and blocked the possibility of advance. And even yet, rigid deduction from the most hare-brained premiss, if the chain of deduction is sufficiently intricate, seems to possess fascinations over a verifiable induction, with many minds.

And now, if any ask, "*cui bono*" to the scientist, these philosophical inquiries and intricacies when he has the vast field of unexplored data still before him to occupy him, I answer, the queries of Philosophy are not only the main-spring and final cause of science (her first fruitful daughter and handmaid), but they, consciously or unconsciously, dominate the methods and results of science herself. Each investigator, even though in the domain of the most abstract of the sciences, postulates more philosophy than he is aware of; and with so much the more danger to final accomplishment if he assumes his philosophical basis without examination. It is the errors of giant minds that are dangerous, by their ponderosity. The infallibility of the master, Aristotle, seemed to make investigation useless, until the rise of parallel giants, like Galileo and Copernicus, stimulated a new conflict of opinion. And Descartes, though harmless from all his productions within the metaphysical domain, is dangerous by his very eminence and originality in science, which gives vogue and currency to his monumental errors. Although acquainted with the true law of motion, his scheme of matter evolved from consciousness would forbid all exhibition thereof. A grand geometer, he erected a scaffold for scaling immensity, and with unparalleled penetration perceived how a purely ideal logic, if general, would represent truth in a wholly dissimilar realm of deduction, if equally general. Strange to say, this grand and useful discovery has become the engine, in nihilistic hands, for overthrowing all the positive knowledge we

possess—the achievements of two thousand years of human effort. Not only geometry—all that has survived to us of philosophical value from the antique world—but the basis of positive dynamics, as handed down from Galileo and Newton and Boscovich and Dalton, are apparently undermined, for all that gives them intellectual value—their certainty—unless an effort be made in the neglected field of philosophy. With strange inconsistency these advocates *par excellence* of the experiential origin of knowledge are found in the same breath promulgating as possible truth matters not only non-experiential, but not representable in ideas derived from or verifiable by experience, and avowedly originating not from inductive generalizations—the only source of knowledge—but in purely deductive processes in the old scholastic way, from logical premises of bald assumption. In a similar way, in the hands of the Greek sophist, language, a good servant, became a vicious master, and made a chaos of all ethical achievement. A remnant of knowledge, fortunately expressed, not in verbal, but diagrammatic logic—geometry—was left, but only to fall now by the hands of similar iconoclasts, armed with more potent destructiveness, in its full flower and fruit of twenty centuries of unmolested growth.

It is time, therefore, to get back to Baconian ground, and while using for its legitimate purposes the magnificent modern machinery of analytical investigation in the field of abstract continuity—extension, motion, duration—not attempt to conjure with it as a source of objective revelation, which no mere machinery can be. A scaffold of *n* dimensions is as useless to the geometer as to the architect. To assume matter as continuous, simply because of the possession of a potent engine for the investigation of continuities, is to repeat the practice of certain quack specialists, who are prone to diagnose nearly every form of disease as a variety of their own peculiar specialty. And to interview the symbols of a mathematical logic for the prime definition of a fundamental objectivity, like force, is to revert to a barren source of knowledge, by an obsolete process in philosophy, and bar all progress in anything but abstract technique.